# Solving Problems Using Problem Solving

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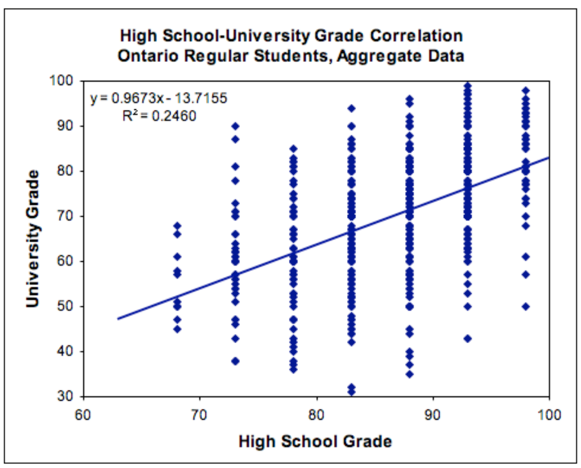
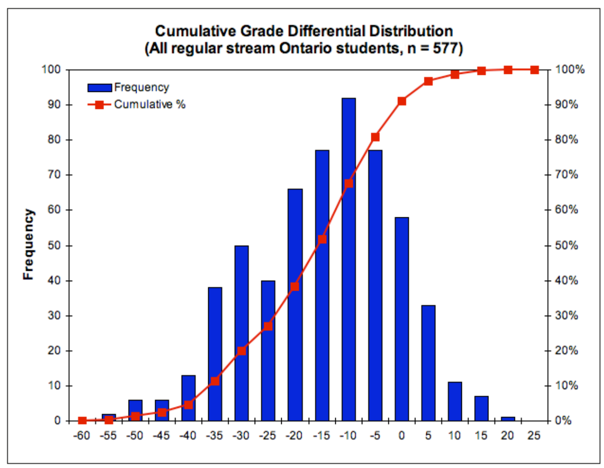
**A big problem:**

- There is **very little correlation** between high school grades and subsequent performance in university, even when the course content appears to be simple 'review' of the high school curriculum...
- 25% of students see their grade **drop by 30-60 percentage points** in first year undergraduate chemistry compared to high school...



**Self-Test/Assessment Questions**

The following are all taken from 1st and 2nd-year university multiple-choice student self-assessment tests. The distractors in some of the questions were deliberately chosen based on known errors that students commit; this enables highly specific feedback to be provided to the students based on their choices for each question. Such an approach is not recommended for high-stakes tests or exams!

1. Given the expression A/B = X/Y, then B is equal to:
    a. XY/A
    b. AY/X
    c. X/AY
    d. Y/AX

2. When correctly expressed in SI units, a density of 1.23 $g/cm^3$ is:
    a. 1.23 x $10^{-3}$ $g/m^3$
    b. 1.23 x $10^{-3}$ $kg/m^3$
    c. 1.23 $g/m^3$
    d. 1.23 x $10^3$ $kg/m^3$

3. Consider the following balanced chemical reaction:
    $$2\ MnO_4^- + 16\ H^+ + 15\ I^- \rightarrow 2\ Mn^{2+} + 5\ I_3^- + 8\ H_2O$$
    What volume of 0.0525 M iodide would be required to exactly react with 20.0 ml of 0.0125 M permanganate?
    a. 0.63 ml
    b. 4.76 ml
    c. 35.7 ml
    d. 84.0 ml

4. A solution of known iodine concentration may be prepared by mixing solutions of iodate and iodide under acidic conditions:
    $$a\ IO_3^- + b\ I^- + c\ H^+ \rightarrow p\ I_2 + q\ H_2O$$
    When correctly balanced, the stoichiometric coefficients in this reaction equation are:
    a. $a = 1, b = 1, c = 6, p = 1, q = 3$
    b. $a = 1, b = 5, c = 6, p = 3, q = 3$
    c. $a = 3, b = 3, c = 6, p = 3, q = 3$
    d. $a = 5, b = 1, c = 6, p = 1, q = 5$

The following two questions appear back-to-back on the University of Guelph 1st-year chemistry student evaluation test. The results obtained are consistent year-over-year...

5. The geometry of a water molecule is:
    a. angular or bent
    b. linear
    c. pyramidal
    d. tetrahedral

6. The geometry of a molecule in which the central atom is bonded to 2 H atoms and has 2 lone pairs is:
    a. angular or bent
    b. linear
    c. pyramidal
    d. tetrahedral

Notes: question 1 is typical of every chemistry diagnostic exam or test of formal (logical) thinking development; the ability to do basic algebraic manipulations of equations – and especially those involving ratios – is essential to doing well in physical sciences. The following tables summarises student response rates for each option in the preceding multiple-choice questions. Correct answers are in bold font.

| Question | a | b | c | d | Comment |
|---|---|---|---|---|---|
| 1 | 7.9% | **68.4%** | 18.4% | 5.3% | |
| 2 | 4.8% | 28.6% | 4.8% | **61.9%** | (b) inverted conversion factors |
| 3 | 7.0% | 18.6% | **72.1%** | 2.3% | (a) inverted coefficients<br>(b) omitted coefficients<br>(d) inverted concentrations |
| 4 | 51.2% | **41.9%** | 7.0% | 0% | (a) balanced mass only<br>(c) balanced charges only |
| 5 | **88.8%** | 4.5% | 4.6% | 2.1% | |
| 6 | 15.9% | **53.0%** | 13.9% | 17.2% | |

## Threshold Concepts and Troublesome Knowledge

Threshold Concepts (Meyer & Land) [Refs. 6 & 9]:

> "[Are] akin to a portal, opening up a new and previously inaccessible way of thinking ... without which the learner cannot progress"

Troublesome Knowledge (Perkins) [Ref. 7; see also ref. 6]:

> "Some students will resort to rote memory and routine procedures ... They will try to learn enough ... to pass the test without developing any real insider feel. And pass they may, ending up with *knowledge troubled by partial and brittle understandings*..."

- Ritual: names and dates, arithmetic procedures
- Inert: vocabulary, concepts (no transfer of learning between subjects)
- Conceptually difficult: naive/alternate conceptions (misconceptions) arising from concepts that contradict "common sense" (see below)
- Foreign or alien: no connection to learner experience or frame of reference *e.g.* eight-tracks, rotary dial phones, different society/cultural values & norms...
- Tacit: personal and implicit; contains subtle distinctions *e.g.* equal temperament in music (tuning of piano, concept of scales)

# Alternate Conceptions
# (Naive conceptions, misconceptions)

Alternate conceptions arise from (Kind, 2004; Talanquer, 2006; Taber & Garcia Franco, 2009):

- "Common sense" reasoning
- Intuitive thinking
- Personal experience
- Mental shortcuts ("cognitive misers" – Keith Stanovich, 2009)
- Misuse of analogies (inappropriate or limits not stated)
- Misunderstanding of the nature of scientific models
- The way we teach and grade assessments

"Many of students' alternative conceptions in chemistry seem to result from the confident and impulsive application of a crude, incomplete, limited, and superficial explanatory framework about chemical substances and phenomena. This knowledge system ... creates the illusion of explanatory depth: **students believe that they understand more than they actually do**."

(Talanquer, 2006, emphasis added)

*An example of a subtle distinction is the dual use of the = symbol to represent 'can be calculated' (equals) and 'is equivalent to'; this may be the source of some student confusion over stoichiometric calculations and unit conversions, depending on how they have been taught! Consider the following example from Herron's paper:*

There are six times as many students as professors at a particular university. This can be expressed mathematically as:

a) $6S = P$ "six students *are equivalent to* one professor"
b) $S = 6P$ "the number of students *equals* six times the number of professors"

*Remember that, if one thing is **defined** to be exactly equivalent to another, we can (and arguably should) use the ≡ symbol:* 1 atm ≡ 101.325 kPa ≡ 760 mmHg; 2 mol NaOH ≡ 1 mol $H_2SO_4$ *etc.*

*In stoichiometric calculations it is important to remember that, whether you use the mole-ratio or factor-label methods, the conversion factors stem from the **same** fundamental chemical principles: **conservation of mass and definite proportions**. Discussion about the implications of these should lead to the conclusion that the stoichiometric mole ratio for **any** pair of substances in a chemical reaction is **always** equal to the ratio of the reaction coefficients:*

$$a\,\mathrm{A} + b\,\mathrm{B} \rightarrow p\,\mathrm{P} + q\,\mathrm{Q}$$

$$\therefore \frac{n_A}{n_B} = \frac{a}{b}, \frac{n_A}{n_P} = \frac{a}{p}, \frac{n_A}{n_Q} = \frac{a}{q}, etc. \qquad [1]$$

*Students should be encouraged to start **any** stoichiometric or limiting reagent calculation by writing this relationship for the pair of substances required **before** any conversions to mass,* *concentration, or pressure are applied. It is good preparation to encourage substitution and rearrangement before calculation, as students need as much practice with this in a chemistry context as possible. So equation [1] can be modified using any of the following:*

$$n = \frac{m}{M_m},\ n = cV,\ n = \frac{PV}{RT}$$

*Example 1: stoichiometric mass of P formed from mass of A*

$$\frac{n_P}{n_A} = \frac{p}{a}$$

$$\therefore n_P = n_A \times \frac{p}{a}$$

$$n = \frac{m}{M_m}$$

$$\therefore m_P = m_A \times \frac{M_m(P)}{M_m(A)} \times \frac{p}{a}$$

*Example 2: given moles of A and B, determine which one is the limiting reagent*

$\frac{n_A}{n_B} = \frac{a}{b}$ – if LHS > RHS, then B is limiting reagent, *etc.*

*Example 3: titration calculation*

$$\frac{n_A}{n_B} = \frac{a}{b}$$

$$\therefore n_A = \frac{a}{b} \times n_B$$

$$n = cV$$

$$\therefore c_A V_A = \frac{a}{b} c_B V_B$$

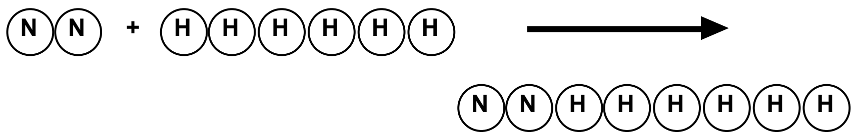
$$\therefore c_A = c_B \times \frac{a}{b} \times \frac{V_B}{V_A}$$

*Note that the above inherently avoids confusing dilution with titration! In all cases, these are mathematically equivalent to the way the same calculations are expressed in textbooks, but each makes the origin of the conversion factors explicit, and avoids any confusion between equals and equivalent!*

*One way to probe "conceptual" (understanding) versus "algorithmic" (ritual or procedural) thinking is to ask paired questions. The first is a routine calculation, while the second probes students' understanding of what they have just been asked to do. The following is taken from Mary Nakhleh's paper [Ref. 5]:*

a) Balance the following equation for the production of ammonia: $N_2 + H_2 \rightarrow NH_3$
b) Represent the balanced reaction using circles with letters in the centre to depict the atoms:

*Here is what one student drew:*




*A common misconception, possibly stemming from the way we describe splitting a substance into smaller and smaller units, is that individual atoms have the **same** physical properties as their macroscopic substance, e.g. Cu atoms have the same properties as bulk metallic copper; this is plainly false! The following questions are adapted from Ref. 1:*

A metallic wire has the following properties: a) conducts electricity; b) brown colour; c) density of 8.93 g/cm$^3$; d) malleable and ductile; e) expands on heating. Suppose you could isolate one single atom from the metallic wire: which of the above properties would it have?

*None of the above! For density, remember that there are voids between the atoms even with the closet packing arrangement.*

The wire is heated in an evacuated vessel until it completely evaporates. The resulting gas has the following properties: a) compressible; b) expands on heating; c) pungent odour; d) yellow colour; e) attacks plastics. Suppose you could isolate one single atom from this gas: which of the above properties would it have?

*You could debate about c and e, until you remember that we are talking about **single** atom here...*

*The following are all examples taken from concept tests. Note particularly the gas law example, which shows how you can use the same question to develop and subsequently test for specfic concepts. In addition, there are many conceptual problems and a "Chemistry Concepts Inventory" (similar to the physics Force Concepts Inventory) available from the J. Chem. Ed. question bank site: http://jchemed.chem.wisc.edu/JCEDLib/QBank/collection/index.html*

**Mole Concept Test** (From Duncan & Johnstone, *Education in Chemistry*, 1973, 213-214)

1. Given that $Mg + S \rightarrow MgS$, what mass of Mg would react completely with 32 g of S?
   a) 12 g
   b) 24 g
   c) 32 g
   d) 56 g

2. Given that $2\ NaOH + H_2SO_4 \rightarrow Na_2SO_4 + 2\ H_2O$, how many moles of $H_2SO_4$ are required to react with 1 mole of NaOH?
   a) ½ mole
   b) 1 mole
   c) 2 moles
   d) 4 moles

3. Given that $2\ SO_2 + O_2 \rightarrow 2\ SO_3$, what mass of $SO_2$ would react with 32 g of $O_2$?
   a) 32 g
   b) 64 g
   c) 96 g
   d) 128 g

4. Given that $C + O_2 \rightarrow CO_2$, what mass of $O_2$ is required to react with 3 g of carbon?
   a) 8 g
   b) 16 g
   c) 64 g
   d) 128 g

5. What is the correctly balanced form of the equation $Al + O_2 \rightarrow Al_2O_3$?
   a) $2\ Al + O_3 \rightarrow Al_2O_3$
   b) $Al_2 + 3\ O \rightarrow Al_2O_3$
   c) $4\ Al + 3\ O_2 \rightarrow 2\ Al_2O_3$
   d) $Al_4 + 6\ O \rightarrow 2\ Al_2O_3$

6. Given that $N_2 + H_2 \rightarrow NH_3$, how many moles of $H_2$ would react completely with 1 mole of $N_2$?
   a) 1 mole
   b) 2 moles
   c) 3 moles
   d) 4 moles

7. A molar solution of HCl contains
   a) 1 mole of HCl dissolved in 1 mole of water
   b) 1 mole of HCl dissolved in 1 litre of water
   c) 1 mole of HCl dissolved in 1 litre of solution
   d) 1 mole of water dissolved in 1 litre of HCl

8. Which solution of NaCl is most concentrated?
    a) 200 mL of a solution containing 2 moles of dissolved NaCl
    b) 500 mL of a solution containing 4 moles of dissolved NaCl
    c) 750 mL of a solution containing 8 moles of dissolved NaCl
    d) 1000 mL of a solution containing 6 moles of dissolved NaCl

9. Which of the following solutions of HCl is most concentrated?
    a) 500 mL of 2 M HCl
    b) 1000 mL of 3 M HCl
    c) 300 mL of 4 M HCl
    d) 800 mL of 5 M HCl

10. Which of the following solutions contains the most NaCl?
    a) 500 mL of 2 M NaCl
    b) 1000 mL of 3 M NaCl
    c) 250 mL of 4 M NaCl
    d) 200 mL of 5 M NaCl

Notes and comments on questions 1–10 from the original study of Scottish high school students: Facility value (FV) is the percentage answering the question correctly; discrimination power (DP) is the difference in FV values between the top and bottom third of the students when ranked by overall score (for large classes, upper and lower quartiles can be used instead).

| Num. | Answer | FV(%) | DP(%) | Comment |
|---|---|---|---|---|
| 1 | b | 80 | 42 | |
| 2 | a | 51 | 55 | Deviation from 1:1 stoichiometry causes problems |
| 3 | d | 37 | 57 | Similar comment |
| 4 | a | 72 | 41 | |
| 5 | c | 55 | 39 | Balancing equations problematic |
| 6 | c | 37 | 29 | Almost half chose answer (a) |
| 7 | c | 55 | 22 | Students unaware of finite volume of *solute* in solvent! |
| 8 | c | >66% | n/a | |
| 9 | d | 42 | 15 | Low DP implies even best students making mistakes |
| 10 | b | 39 | 10 | Most ignored the volume in the problem |

Additional Note:

Half of 2$^{nd}$-year undergraduates in an analytical chemistry course incorrectly chose b for question 7 on a self-assessment test

Additional conceptual questions can be found on the J. Chem. Ed. Question Bank web site at:

http://jchemed.chem.wisc.edu/JCEDLib/QBank/collection/CCInventory/index.html

## Structured, Diagnostic, and Conceptual questions

This question is an example of conceptual vs. algorithmic ("algebraic") problem solving, and can be asked in one of three ways. Source: Lillian Bird, *J. Chem. Ed.*, **2010**, *87(5)*, 541-546.

*Structured example:*

A flask is first evacuated, then filled with 0.200 g of methane gas ($CH_4$) and stoppered.

a) The molecular mass of methane is 16.0 g/mol. How many moles of methane are contained in the flask?
b) The flask containing the methane is held at a constant temperature of 298 K. What is the pressure if the flask has a volume of 1.00 L? The value of the gas constant $R$ = 0.0821 atm L/(mol K).
c) A second flask is filled with 0.200 g of carbon dioxide gas ($CO_2$, molecular mass of 44.0 g/mol). How many moles of carbon dioxide are contained in the second flask?
d) The flask containing the carbon dioxide is also held at a constant temperature of 298 K. What is the pressure in this flask if it also has a volume of 1.00 L? The value of the gas constant $R$ = 0.0821 K/(mol L).
e) Compare your answers to b) and d). How does the pressure in each flask relate to the molecular weight of the gas within it, given that both have the same volume and are at the same temperature?
f) Use this information to predict whether a 1.00 L flask containing 0.200 g of nitrogen gas ($N_2$, molecular weight 28.0 g/mol) at 298 K would have a higher or lower pressure than the flask containing the carbon dioxide. Explain your reasoning.

*Diagnostic example:*

Individual 0.200 g samples of each of the following gases were placed in four separate 1.00 L stoppered flasks at 298 K. In which flask do you expect the gas to exert more pressure? Explain your answer.

| **Flask:** | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Gas in flask:** | $CH_4$ | Ne | $N_2$ | $CO_2$ |
| $\boldsymbol{M_m}$ **(g/mol)** | 16.0 | 20.2 | 28.0 | 44.0 |

Lillian Bird notes that of 106 students: 36% calculated the pressure for each gas and justified their answer solely on these values; 42% similarly did all the calculations but then derived the principle and used that as justification; 26% gave the correct answer solely in terms of principles.

*Concept text example:*

Four identical sealed containers are each filled with a different gas (as indicated below) until each contains exactly the same mass. If all four are held at the same temperature, which flask contains gas at the greatest pressure?

This can be provided with or without the same table as before; students can only provide a solution if they can manipulate the concepts and equations *without* the benefit of a concrete calculation, since the mass, volume, and temperature are never stated!

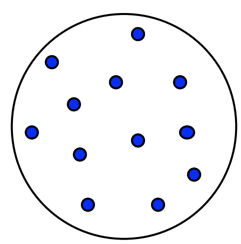
## Conceptual *versus* Algorithmic (Algebraic) Thinking

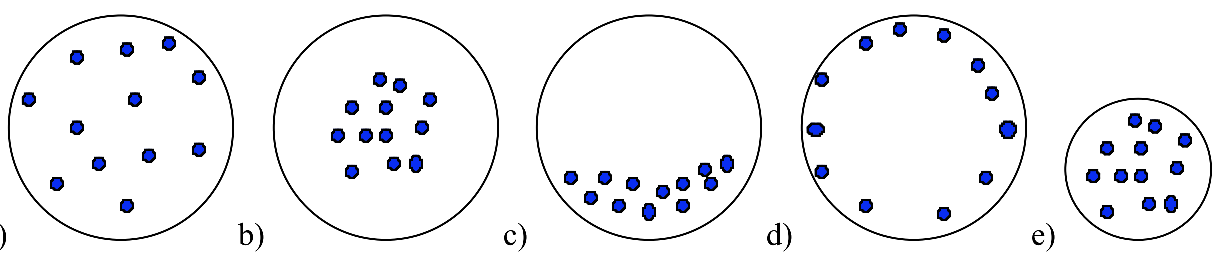
Mary Nakhleh and others have adopted a diagnostic approach using *paired* questions: the first usually requires a fairly straight–forward calculation that can be done simply by following a memorized procedure (algorithmic or algebraic thinking); the second explores a student's *conceptual* understanding of the same phenomenon. A number of the alternate conceptions test questions use the same structure.

*Gas laws example:*

1. 0.100 mole of hydrogen gas occupies 600 mL at 25 °C and 4.08 atm. If the volume is held constant, what will be the pressure of the sample of gas at –5 °C?
    a) 2.98 atm
    b) 3.67 atm
    c) 4.08 atm
    d) 4.54 atm
    e) 6.00 atm

2. The following diagram represents a cross-sectional view of a rigid sealed tank filled with hydrogen gas at 20 °C and 3 atm pressure. The dots represent the distribution of hydrogen molecules within the tank. Which of the diagrams (a) through (e) illustrates one possible distribution of the hydrogen gas molecules in the tank if its temperature is lowered to –5 °C? The normal boiling point of hydrogen is –252.8 °C.





a) b) c) d) e)

*Limiting reagent example:*

1. Which is the limiting reagent when 2.0 mol of $CO_2$ reacts with 2.0 mol of $S_2$ to form COS and $O_2$?
    a) $CO_2$ b) $S_2$ c) COS d) $O_2$ e) none are limiting

2. Atoms of three different elements are represented by ⊗, ⊕, and ∅. Which is the limiting reagent when two ⊗⊗ molecules and two ∅∅⊕ molecules react to form ⊗∅⊕ and ∅∅?
    b) ⊗⊗ b) ∅∅⊕ c) ⊗∅⊕ d) ∅∅ e) none are limiting

**Note:** The terminology applied to these examples follows a simplified version of Bloom's taxonomy, consisting of **R**ecall, **A**lgorithmic (or Algebraic), and **C**onceptual questions. This provides a much easier means of checking whether or not a test or exam meets the instructor's intended goals as far as the types of evaluation. For a typical 1$^{st}$- or 2$^{nd}$-year undergraduate course, for example, the marks might be distributed amongst the three categories of question as 33% R, 33% A, and 33% C.

## Problem solving group activity:

- Get into groups of about 3 or 4, and have one person volunteer to act as an observer. Turn to one of the following problems and solve it as a group.

- The observer should take notes on how the group goes about solving the problem, any difficulties they encounter, and how they resolve them.

  *Note: if the problem is one you have seen before, volunteer to be the observer!*

**Observer Notes:**

## The Waterfall Problem

The Horseshoe Falls are 49 m high. Assuming that all the potential energy of the water is converted into heat, how much warmer is the water at the bottom of the falls than at the top? Comment on the magnitude of your answer. The flow of water over the falls is reduced at night as more is diverted through the hydroelectric generating station. What affect will this have? Give reasons for your answers.

Relevant Data:

> Potential energy $E_p = mgh$ where $m$ is mass (kg), $g = 9.81$ m s$^{-2}$, and $h$ is height ($m$); the specific heat capacity of water $s = 4.179$ J/(g °C).

## The Pizza Problem

We all know that if you try and eat pizza too soon after it comes out of the oven you can burn your mouth. Is this because of the crust, the cheese, or the tomato sauce? Use your knowledge of the different phases of matter, kinetic molecular theory, and thermochemistry to justify your answer.

Some relevant data:

- Assume that a 50 g slice of pizza consists of 25 g crust, 20 g cheese, and 5 g of sauce. Approximate values of the specific heat capacity are: Cheese = 3.0 J/(g °C); Crust = 2.0 J/(g °C); Sauce = 4.0 J/(g °C). The heat capacity of water = 4.2 J/(g °C).
- Pizza is cooked at ~ 450 °F (230 °C). Assume that, by the time it reaches your table, the pizza has reached a uniform temperature of about 150 °C. The soft tissue on the inside of your mouth (which has a very high water content) is 37 °C. Cheese melts at around 40 °C. The latent heat of fusion of milk fat $\Delta H_{fus}$ = 84 kJ/kg.

## The Water and Wine Problem

You have a glass of water and a glass of wine. Assume that both are pure, homogeneous substances. (If it helps, consider the wine to be pure ethanol!)

1. Transfer exactly one teaspoon from the glass of water to the glass of wine and mix thoroughly.
2. Transfer exactly one teaspoon of this contaminated wine to the glass of water and mix thoroughly.

Consider the amount of water in the glass of wine, and the amount of wine in the glass of water: Which of the following statements is *true*?

a) The amount of water in the wine is greater than the amount of wine in the water
b) The amount of wine in the water is greater than the amount of water in the wine
c) The amount of water in the wine is equal to the amount of wine in the water

## The Xenon Fluoride Problem

A sample of a compound comprising only xenon and fluorine was confined to a bulb with a pressure of 24 torr. Hydrogen was added to the bulb until the pressure was 96 torr. Passage of an electric spark through the mixture produced Xe and HF. After the HF was removed by reaction with solid KOH, the final pressure of xenon and unreacted hydrogen in the bulb was 48 torr. What is the empirical formula of the xenon fluoride in the original sample?

# Problem Solving

John Hayes: "Whenever there is a gap between where you are now and where you want to be, and *you don't know how* to find a way to cross that gap, you have a problem" (emphasis added; as cited by Bodner, 2003).

Polya's model of problem solving (ok for routine exercises but...) *C.f.* "GRASS"

- Understand the problem ← *if you understood it, it wouldn't be a problem!*
- Devise a plan
- Carry out the plan
- Look back

The Anarchistic Route, or "How an *expert* solves problems" (Wheatley, as cited by Bodner):

- **Read the problem**
- Now read the problem again
- Write down what you hope is the relevant information
- **Draw a picture**, make a list, or write an equation or formula to help you begin to understand the problem
- Try something
- Try something else
- See where this gets you
- Read the problem again
- Try something else
- See where this gets you
- Test intermediate results to **see whether you are making any progress** toward an answer
- Read the problem again
- When appropriate, strike your forehead and say, "son of a..."
- Write down 'an' answer (not necessarily 'the' answer)
- **Test the answer to see if it makes sense**
- Start over if you have to, celebrate if you don't

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17. Students’ and Teachers’ Conceptions and Science Education, final version compiled by Reinders Duit, University of Kiel Institute for Science Education, 2009. (Includes all branches of secondary school science) Available for download from http://www.ipn.uni-kiel.de/aktuell/stcse/

# The Waterfall Problem

The Horseshoe Falls are 49 m high. Assuming that all the potential energy of the water is converted into heat, how much warmer is the water at the bottom of the falls than at the top? Comment on the magnitude of your answer. The flow of water over the falls is reduced at night as more is diverted through the hydroelectric generating station. What affect will this have? Give reasons for your answers.

Relevant Data:

Potential energy $E_p = mgh$ where $m$ is mass (kg), $g$ = 9.81 m s$^{-2}$, and $h$ is height ($m$); the specific heat capacity of water $s$ = 4.179 J/(g °C).

## Solution:

First, we need to know how much energy is available to be converted into heat. This is the potential energy associated with the water being at the top of falls. Since we don't know how much water is going over the falls, the simplest approach is to calculate for unit mass:

$$E_p = mgh = 1 \times 9.81 \times 49 \text{ kg} \cdot \text{m s}^{-2} \cdot \text{m} = 481 \text{ kg m}^2 \text{ s}^{-2} \text{ or } 481 \text{ J}$$

Next, we need to convert this to a change in temperature. Remember that *changes in state* are always (final – initial), so $\Delta T = (T_{final} - T_{initial})$. We assume that all the potential energy is converted into heat, so that:

$$E_p = q = sm\Delta T \therefore \Delta T = \frac{E_p}{sm} = \frac{481}{4.179 \times 1 \times 10^3} \; \frac{\text{J}}{\text{J g}^{-1}\ {}^\circ\text{C}^{-1} \cdot \text{kg} \cdot \text{g kg}^{-1}} = \mathbf{0.12\ {}^\circ C}$$

This is a small but reasonable value, given the assumptions made. When the flow of water is reduced overnight, the potential energy of each kilogram of water at the top of the falls is still the same, so the warming will be the same. Heat transfer to the surroundings will obviously be different at night, but we did not consider that in our initial calculation either.

## Comments:

Some students don't even know where to start this problem, as they are unused to connecting concepts from different parts of the course (let along different courses!) Many make a common mistake with units and predict an extremely large temperature difference that is in excess of water's normal boiling point. Of these, some state that it "looks wrong" but many don't comment at all.

In discussing this question, point out that students need to identify the core concepts first, rather than simply 'plugging and playing' with the first equation that seems relevant. It is also very helpful to emphasize unit analysis for physical chemistry problems such as this, as errors can often be caught here. When presenting the solution, be sure to state the *thought process* at each step before the actual calculation! Finally, since the height is only given to 2 s.f., the final answer can only be stated to 2 significant figures.

## The Pizza Problem

We all know that if you try and eat pizza too soon after it comes out of the oven you can burn your mouth. Is this because of the crust, the cheese, or the tomato sauce? Use your knowledge of the different phases of matter, molecular-kinetic theory, and thermochemistry to justify your answer.

Some relevant data:

- Assume that a 50 g slice of pizza consists of 25 g crust, 20 g cheese, and 5 g of sauce. Approximate values of the specific heat capacity are: Cheese = 3.0 J/(g °C); Crust = 2.0 J/(g °C); Sauce = 4.0 J/(g °C). The heat capacity of water = 4.2 J/(g °C).
- Pizza is cooked at ~ 450 °F (230 °C). Assume that, by the time it reaches your table, the pizza has reached a uniform temperature of about 150 °C. The soft tissue on the inside of your mouth (which has a very high water content) is 37 °C. Cheese melts at around 40 °C. The latent heat of fusion of milk fat $\Delta H_{fus}$ = 84 kJ/kg.

There are, in fact, two factors involved: the heat capacity (how much energy is stored in the materials and is available to be transferred to your cheek), and the thermal conductivity (how rapidly the heat energy can be transferred.) Since the heat energy is stored as random molecular kinetic motion, these are related. The key difference between crust, cheese, and sauce is the *water content*, which will generally tend to dominate the heat capacity. In other words, the higher the water content, the greater amount of heat that can be stored for the same rise in temperature. This is enhanced when a phase change (*i.e.* the cheese melting) occurs, since the energy added or lost during the phase change does *not* change the temperature!

### Solution:

Calculate the heat energy available in each ingredient that can be transferred to your cheek. You can either (a) assume an initial starting temperature (either 21 or 25°C would be reasonable) to compare the total amount of energy stored by the time the pizza is a uniform 150 °C, or (b) simply look at how much heat energy can be transferred *i.e.* $\Delta T$ = (150 – 37) = 113 °C.

Since the temperature of the pizza is above the cheese melting point and that of your cheek below, include the amount of energy required to melt the cheese, *i.e.*:

$$q_{cheese} = q_{heat} + q_{melt} = sm\Delta T + (84 \times 20)$$

Note that units of kJ/kg are the same as J/g, so the conversion factors cancel! Ultimately, the most liquid component will transfer its heat the fastest, as this is accomplished by molecular collisions (refer back to molecular kinetic theory), so it is in fact the sauce that burns first, even though there is less of it. Having said that, the cheese will likely cause greater burning if you keep it in contact with your lip, tongue or cheek too long. So always have pop with your pizza!

# The Water and Wine Problem

***Note**: this problem was originally proposed by a philosopher, in which the question and proof involved discussion of the **purity** of the water in the wine and vice versa. Dudley Herron changed this to amount when he presented it (see ref. 8 and book 3). I've used volume here, although this perhaps helps prevent students from slipping into the most common mistake, which is to assume that we have to calculate concentrations. You may want to try both wordings!*

You have a glass of water and a glass of wine. Assume that both are pure, homogeneous substances. (If it helps, consider the wine to be pure ethanol!)

1. Transfer exactly one teaspoon from the glass of water to the glass of wine and mix thoroughly.
2. Transfer exactly one teaspoon of this contaminated wine to the glass of water and mix thoroughly.

Consider the amount of water in the glass of wine, and the amount of wine in the glass of water: Which of the following statements is *true*?

a) The amount of water in the wine is greater than the amount of wine in the water
b) The amount of wine in the water is greater than the amount of water in the wine
**c) The amount of water in the wine is equal to the amount of wine in the water**

## Solution:

Most people – including university faculty members – get this wrong, and many still don't believe the answer after seeing the proof until they've worked it for themselves. So this really is a challenging problem, and I'd save it for your most advanced (or most self-assured!) students. I would also note that I found the proof, as originally presented, extremely hard to follow and therefore difficult to accept. Here goes...

(i) Start with two containers, (1) and (2), containing pure A (water) and pure B (wine), so that:

$$V_1 = V_A^\circ \text{ and } V_2 = V_B^\circ$$

(ii) Let the volume transferred from A to B in the first step be $V_{1,2}$. We now have:

$$V_1' = V_A^\circ - V_{1,2} \text{ and } V_2' = V_B^\circ + V_{1,2}$$

(iii) The volume transferred from (2) back into (1) contains a small fraction of A ($\Delta V_A$) from $V_{1,2}$. Both glasses now have their original liquid volumes; neither contains pure A or pure B:

$$V_1'' = V_A^\circ - V_{1,2} + V_{2,1} \text{ and } V_2'' = V_B^\circ + V_{1,2} - V_{2,1}$$

(iv) How much $A$ is now in $B$ in glass (2)? $V_{A,B} = V_{1,2} - \Delta V_A \therefore\ V_{1,2} = V_{A,B} + \Delta V_A$
How much $B$ is now in $A$ in glass (1)? $V_{B,A} = V_{2,1} - \Delta V_A \therefore\ V_{2,1} = V_{B,A} + \Delta V_A$

(v) But $V_{1,2} = V_{2,1}$ therefore equating and rearranging from (iv) gives:

$$V_{A,B} + \Delta V_A = V_{B,A} + \Delta V_A \ \therefore\ V_{A,B} = V_{B,A} \ \textbf{(Q.E.D)}$$

## The Xenon Fluoride Problem

A sample of a compound comprising only xenon and fluorine was confined to a bulb with a pressure of 24 torr. Hydrogen was added to the bulb until the pressure was 96 torr. Passage of an electric spark through the mixture produced Xe and HF. After the HF was removed by reaction with solid KOH, the final pressure of xenon and unreacted hydrogen in the bulb was 48 torr. What is the empirical formula of the xenon fluoride in the original sample?

*In the article giving this problem, it was noted that many chemistry professors other than those routinely teaching 1st-year would figure out the formula ($XeF_4$) and only then realise that it was "just an empirical formula question"! It is completely possible to solve this without needing the temperature, volume, gas constant, and pressure conversion factors, as long as it is assumed that all pressures were measured under the same conditions. Try it...*